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# DYNAMICS OF THE DISTURBANCES OF A WEAKLY IONIZED PLASMA UNDER CONDITIONS OF THE CHAMBERS OF THERMAL ENERGY CONVERTERS

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*Relations between the characteristics of disturbing acoustic effects and the parameters of state of a medium which correspond to the conditions of the development of charge instabilities in the low-temperature plasma of combustion products in chambers of thermal energy converters are determined in dimensionless form. These relations are confirmed by the results of a numerical experiment.*

This work is a continuation of [1–3] and is devoted to investigation of the processes of formation of charge instabilities in weakly ionized working bodies (combustion products (CP)) of thermal energy converters.

I. We consider a CP flow — flow of a weakly ionized plasma consisting of electrically and gasdynamically interacting neutral a, electronic e, and ionic i components. Due to a weak ionization the magnetic interaction of them is disregarded. A one-dimensional problem is considered.

The description is based on an electrogasdynamic system [1]. The system is represented in dimensionless form by the equations of motion of charge carriers, continuity of the flow, and conservation of charge:

$$n_e\left(\frac{\partial}{\partial t}+U_e\nabla\right)U_e+n_e\,\varphi_e\,(U_e-U_a)+\frac{m_a}{m_e}\nabla\,(n_eT)-\frac{m_a}{m_e}n_eE=0\,,$$

$$n_i\left(\frac{\partial}{\partial t}+U_i\nabla\right)U_i+n_i\,\varphi_i\,(U_i-U_a)+\frac{m_a}{m_i}\nabla\,(n_iT)+\frac{m_a}{m_i}n_iE=0\,, \qquad (1)$$

$$\frac{\partial}{\partial t}\rho+\nabla\,(\rho U)=0\,,\quad \frac{\partial}{\partial t}q+\nabla Q=0\,.$$

The following parameters are used to reduce the system to dimensionless form: $\tilde{U}^{(0)}_{T_a}=\sqrt{8k\tilde{T}/\pi m_a}$, $m_a$, $n^0_a$, $L$, $S_{(i,e)a}$, and $\lambda_{i,e}$.

The dimensionless quantities are as follows: $x=\tilde{x}/L$, $t=\tilde{t}\tilde{U}^{(0)}_{T_a}$, $U_{e,i,a}=\tilde{U}_{e,i,a}/\tilde{U}^{(0)}_{T_a}$, $\partial_t=\tilde{\partial}_t/\tilde{U}^{(0)}_{T_a}$, $\nabla=\tilde{\nabla}L$, $p=\tilde{p}/(\tilde{n}^{(0)}_a\,\tilde{U}^{(0)}_{T_a})$, $T=k\tilde{T}/(m_a[\tilde{U}^{(0)}_{T_a}]^2)$, $\rho=p/T(1+\eta)$ is the density, $U_s=\left[\frac{m_a}{m_e}\eta U_e+\alpha U_i+(1-\alpha)\,U_a\right]/\left(\frac{m_a}{m_e}\eta+1\right)\cong U_a$ is the mass velocity, $n_{e,i}=\tilde{n}_{e,i}/\tilde{n}^{(0)}_a$ is the concentration of carriers, $q=(\alpha-\eta)\,\rho$, $Q=(\alpha U_i-\eta U_e)\,\rho$, $\varphi_{i,e}=LS_{(i,e)a}n^{(0)}_a\sqrt{m_a/m_{i,e}}=\sqrt{m_a/2m_{i,e}}\,L/L_{i,e}$ is the coefficient of gasdynamic (viscous) scattering of carriers on the neutral component, $E=E_0+E_{ind}$ is the strength of the longitudinal electric field where the dimensionless induced component is determined by the equation div $E_{ind}=\Theta q$, and $\Theta=\dfrac{e^2L^2n^{(0)}_a}{\varepsilon_0 m_a(U_{T_a^{(0)}})^2}$. The parameters $\alpha$ and

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$\eta$ are related to the pressure and the temperature of the Saha equation of ionization equilibrium [1]. The quantities $U_e$, $U_i$, $\alpha$, and $T$ are used as independent variables in linearization.

II. In order to analyze the stability of the flow we study the dynamics of small disturbances of system (1), which is described in the Fourier–Laplace transformation by the equations

$$[z\delta_{\alpha\beta} + e_{\alpha\beta}]\, \delta X_\beta = \delta X_{\alpha 0}\ ; \quad \{\delta X_\alpha\} = \{\delta U_e U_i,\ \delta T,\ \delta\alpha\}\ , \tag{2}$$

where the coefficients $e_{\alpha\beta}$ of the evolution matrix $\varepsilon$ are equal to

$$e_{\beta\beta} = \varphi_\beta + U_\beta^+\ ; \quad A^+ \equiv \mathrm{grad}\, A + ikA\ ; \quad \beta = 1, 2\ (\mathrm{e, i})\ ; \quad \gamma = 3, 4\ (T, \alpha)\ ;$$

$$e_{\beta\gamma} = ik\left[(-1)^{\beta+1}\frac{\Theta}{k^2} + \delta_{\gamma 3} + \frac{T}{n_\beta}\frac{\partial n_\beta}{\partial \gamma}\right] + \frac{\mathrm{grad}\, n_\beta}{n_\beta}\left(\delta_{\gamma 3} - \frac{T}{n_\beta}\frac{\partial n_\beta}{\partial\gamma}\right);$$

$$e_{\gamma\beta} = \frac{(-1)^{\gamma+1}}{\Delta}\left(\frac{\partial q}{\partial\gamma}\frac{\partial U_s}{\partial\beta}\rho^+ - ik\frac{\partial\rho}{\partial\gamma}\frac{\partial Q}{\partial\beta}\right); \quad e_{3\gamma} = \frac{1}{\Delta}\left(\frac{\partial q}{\partial\alpha}\frac{\partial\rho}{\partial\gamma}U_s^+ + \frac{\partial q}{\partial\alpha}\frac{\partial U_s}{\partial\gamma}\rho^+ - ik\frac{\partial\rho}{\partial\alpha}\frac{\partial Q}{\partial\gamma}\right);$$

$$e_{4\gamma} = -\, e_{3\gamma}\,(4 \to 3)\ ; \quad \Delta \equiv \left(\frac{\partial\rho}{\partial T}\frac{\partial q}{\partial\alpha} - \frac{\partial\rho}{\partial\alpha}\frac{\partial q}{\partial T}\right);$$

$k$ and $z$ are the parameters of the Fourier–Laplace transformation.

For analytical investigation of the stability we use the neutrality criterion $N$. It determines the conditions of transition of the system from the stable state to an unstable state [4].

In our case, det $[N]$ = det $[\delta_{\alpha\beta}\varepsilon + Ie^*_{\alpha\beta}]$ = 0 and in the approximation of $\alpha$ $(\eta \ll 1)$

$$\det[N] = \begin{Vmatrix} \varepsilon + Ie^*_{11} & 0 & Ie^*_{13} & Ie^*_{14} \\ 0 & \varepsilon + Ie^*_{22} & Ie^*_{23} & Ie^*_{24} \\ 0 & 0 & \varepsilon + Ie^*_{33} & 0 \\ Ie^*_{41} & Ie^*_{42} & Ie^*_{43} & \varepsilon + Ie^*_{44} \end{Vmatrix} = 0\ . \tag{3}$$

Here $I$ is the unit matrix, $\varepsilon$ is the evolution matrix, and $e^*_{\alpha\beta}$ are the complex-conjugate elements of the matrix $\varepsilon$.

Criterion (3) gives three individual conditions of possible instabilities and their frequencies

$$\text{a)}\ \ \mathrm{grad}\, U_s = \begin{cases} > 0 - \text{system is stable}\ , \\ < 0 - \text{system is unstable}\ , \end{cases} \qquad \omega_{cr} = -\, kU_s \tag{4}$$

is a conditional gasdynamic instability which corresponds to the origination of pressure (temperature) waves in deceleration of the flow; it is possible at any gradients of disturbances;

$$\text{b)}\ \ G_0 + k^2 G_2 + k^4 G_4 + k^6 G_6 = 0\ , \quad \omega_{cr} = -\, ckU_s \tag{5}$$

is a conditional charge-energy instability related to the relation of the Coulomb and thermal energies of carriers; it leads to plasma vibrations;

$$\text{c)}\ \varphi_e(\varphi_i \operatorname{grad} U_s - B) + k^2\left(\frac{T}{1+\xi}C - D\right) = 0\,,\quad \omega_{cr} = -kb_{44} = \frac{-k(U_e + \xi U_i)}{1+\xi}\,, \tag{6}$$

$$B = \sqrt{\frac{m_a}{m_e}}\,\eta\rho\Theta\,;\quad C = \left(U_{i,s}\,\psi\,\frac{\operatorname{grad} n_e}{n_e} + U_{e,s}\,\frac{\operatorname{grad} n_i}{n_i}\right) + \xi\sqrt{\frac{m_a}{m_e}}\,\varphi_e\,;\quad U_{\alpha s} \equiv U_\alpha - U_s\,;$$

$$D = U_{e,s}U_{i,s}\operatorname{grad} U_s + b_{44s}(U_{e,s}\,\varphi_i + U_{i,s}\,\varphi_e)\,;\quad b_{44s} = \frac{U_{e,s} + \xi U_{i,s}}{1+\xi}\,;\quad \psi = \xi\,\frac{m_a}{m_e}\,;\quad \xi = \frac{\eta}{\alpha}$$

is a conditional flow-charge (current) instability dependent on the gradients of disturbances; it is realized in the form of current-density oscillations.

The current instability is caused by inhomogeneities, the difference in the velocities of components, and a change in the temperature. Conditions (5) and (6) have several possible mechanisms of instabilities.

For longwave disturbances ($k \ll 1$), conditions (5) and (6) are transformed to criteria of the form

$$G_0 = \varphi_i\,\varphi_e - \frac{m_a}{m_e}\,\eta\rho\Theta = \begin{cases} > 0 - \text{system is stable,} \\ < 0 - \text{system is unstable,} \end{cases} \tag{7}$$

$$\varphi_i \operatorname{grad} U_s - \sqrt{\frac{m_a}{m_e}}\,\eta\rho\Theta = \begin{cases} > 0 - \text{system is unstable,} \\ < 0 - \text{system is stable ,} \end{cases} \tag{8}$$

obtained earlier in [3] within the limits of a small inhomogeneity.

In the case of shortwave disturbances ($k \gg 1$), condition (5) is not realized ($G_6 \neq 0$). On the contrary, the current instability

$$\left(\frac{T}{(1+\xi)}C - D\right) = \begin{cases} > 0 - \text{system is stable,} \\ < 0 - \text{system is unstable,} \end{cases} \tag{9}$$

caused by the competing effect of stabilizing temperature and destabilizing convection factors is possible. Depending on the gradients and the signs of $U_{e,s}$ and $U_{i,s}$, instability (9) leads to the predominant removal of one charged component. The latter represents one possible mechanism of motive electrification [1, 2].

In the case of finite gradients and low velocities $U_i = U_e = U_s$, from (6) we have the criterion

$$\left(\varphi_i \operatorname{grad} U_s - \sqrt{\frac{m_a}{m_e}}\,\eta\rho\Theta\right) + k^2\left(\xi\sqrt{\frac{m_a}{m_e}}\,\frac{T}{(1+\xi)}\right) = \begin{cases} > 0 - \text{system is stable,} \\ < 0 - \text{system is unstable ,} \end{cases} \tag{10}$$

representing the condition of formation of charge instability determined by the inhomogeneities of the flow, the degree of ionization, the gradients of disturbances, and the temperature. Condition (10) indicates the stabilizing effect of the increase in the gradients and the existence of a limiting (critical) gradient

$$k_{cr}^2 = (\eta\rho\Theta\sqrt{m_a/m_e} - \varphi_i \operatorname{grad} U_s)/(\xi T\sqrt{m_a/m_e}/(1+\xi))\,.$$

It also follows from (10) that when $\operatorname{grad} U_s < 0$, instability is automatically realized for all degrees of ionization and $k < k_{cr}$. When $\operatorname{grad} U_s > 0$, there exists the threshold value of the relative content of the electronic component

TABLE 1. Characteristics of the Flow and the Spectrum of Disturbances of the Plasma Medium of the Chambers of Thermal Energy Converters ($P_0 = 3 \cdot 10^7$ Pa and $T_0 = 3000$ K)

a) $V = 9$ V; $\tilde{t} = 0.122 \cdot 10^{-3}$ sec

| $\tilde{x}$ | $\tilde{U}_e$ | $\tilde{U}_i$ | $\tilde{U}_a$ | $\tilde{\rho}$ | $\alpha$ |
|---|---|---|---|---|---|
| 0.0781 | 5.41 | 5.41 | 5.41 | 33.7 | $0.290 \cdot 10^{-7}$ |
| 0.0938 | 7.27 | 7.27 | 7.27 | 33.7 | $0.288 \cdot 10^{-7}$ |

| $k$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ |
|---|---|---|---|---|
| 0.1 | $-0.079 - 4.748 \cdot 10^{-4} i$ | $0.141 - 0.124i$ | $-15.623 - 1.043 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.124i$ |
| 0.2 | $-0.079 - 9.531 \cdot 10^{-4} i$ | $0.055 - 0.249i$ | $-15.622 - 2.085 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.248i$ |
| 0.3 | $-0.079 - 1.438 \cdot 10^{-3} i$ | $-0.087 - 0.374i$ | $-15.622 - 3.127 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.372i$ |
| 0.4 | $-0.079 - 1.925 \cdot 10^{-3} i$ | $-0.287 - 0.498i$ | $-15.621 - 4.168 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.496i$ |
| 0.5 | $-0.079 - 2.41 \cdot 10^{-3} i$ | $-0.543 - 0.623i$ | $-15.62 - 5.207 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.621i$ |
| 0.6 | $-0.079 - 2.894 \cdot 10^{-3} i$ | $-0.857 - 0.747i$ | $-15.618 - 6.239 \cdot 10^{-3} i$ | $-3.51 \cdot 10^3 + 0.745i$ |
| 0.7 | $-0.079 - 3.378 \cdot 10^{-3} i$ | $-1.228 - 0.872i$ | $-15.616 - 7.262 \cdot 10^{-3} i$ | $-3.51 \cdot 10^3 + 0.869i$ |
| 0.8 | $-0.079 - 3.861 \cdot 10^{-3} i$ | $-1.656 - 0.997i$ | $-15.614 - 8.265 \cdot 10^{-3} i$ | $-3.509 \cdot 10^3 + 0.994i$ |
| 0.9 | $-0.079 - 3.344 \cdot 10^{-3} i$ | $-2.141 - 1.122i$ | $-15.611 - 9.237 \cdot 10^{-3} i$ | $-3.509 \cdot 10^3 + 1.118i$ |
| 1 | $-0.079 - 4.748 \cdot 10^{-4} i$ | $-2.684 - 1.247i$ | $-15.607 - 0.01i$ | $-3.508 \cdot 10^3 + 1.243i$ |

b) $V = 16$ V; $\tilde{t} = 0.122 \cdot 10^{-3}$ sec

| $\tilde{x}$ | $\tilde{U}_e$ | $\tilde{U}_i$ | $\tilde{U}_a$ | $\tilde{\rho}$ | $\alpha$ |
|---|---|---|---|---|---|
| 0.0781 | 5.41 | 5.41 | 5.41 | 33.7 | $0.384 \cdot 10^{-13}$ |
| 0.0938 | 7.27 | 7.27 | 7.27 | 33.7 | $0.380 \cdot 10^{-13}$ |

| $k$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ |
|---|---|---|---|---|
| 0.1 | $-0.079 - 4.809 \cdot 10^{-4} i$ | $-0.028 - 0.187i$ | $-15.622 - 1.337 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.186i$ |
| 0.2 | $-0.079 - 9.647 \cdot 10^{-4} i$ | $-0.113 - 0.373i$ | $-15.622 - 2.674 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.373i$ |
| 0.3 | $-0.079 - 1.448 \cdot 10^{-3} i$ | $-0.255 - 0.56i$ | $-15.622 - 4.008 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.559i$ |
| 0.4 | $-0.079 - 1.931 \cdot 10^{-3} i$ | $-0.453 - 0.746i$ | $-15.62 - 5.34 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.745i$ |
| 0.5 | $-0.079 - 2.414 \cdot 10^{-3} i$ | $-0.708 - 0.933i$ | $-15.619 - 6.664 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.932i$ |
| 0.6 | $-0.079 - 2.897 \cdot 10^{-3} i$ | $-1.019 - 1.12i$ | $-15.617 - 7.976 \cdot 10^{-3} i$ | $-3.51 \cdot 10^3 + 1.119i$ |
| 0.7 | $-0.079 - 3.379 \cdot 10^{-3} i$ | $-1.388 - 1.307i$ | $-15.615 - 9.266 \cdot 10^{-3} i$ | $-3.51 \cdot 10^3 + 1.306i$ |
| 0.8 | $-0.079 - 3.862 \cdot 10^{-3} i$ | $-1.813 - 1.494i$ | $-15.612 - 0.011i$ | $-3.509 \cdot 10^3 + 1.493i$ |
| 0.9 | $-0.079 - 3.345 \cdot 10^{-3} i$ | $-2.295 - 1.681i$ | $-15.609 - 0.012i$ | $-3.509 \cdot 10^3 + 1.68i$ |
| 1 | $-0.079 - 4.828 \cdot 10^{-3} i$ | $-2.835 - 1.869i$ | $-15.605 - 0.013i$ | $-3.508 \cdot 10^3 + 1.867i$ |

c) $V = 9$ V; $\tilde{t} = 0.844 \cdot 10^{-4}$ sec

| $\tilde{x}$ | $\tilde{U}_e$ | $\tilde{U}_i$ | $\tilde{U}_a$ | $\tilde{\rho}$ | $\alpha$ |
|---|---|---|---|---|---|
| 0.0781 | –2.67 | –2.67 | –2.67 | 33.7 | $0.297 \cdot 10^{-7}$ |
| 0.0938 | –3.61 | –3.61 | –3.61 | 33.7 | $0.288 \cdot 10^{-7}$ |

| $k$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ |
|---|---|---|---|---|
| 0.1 | $0.04 + 2.412 \cdot 10^{-4} i$ | $0.141 - 0.545i$ | $-15.504 - 2.295 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 0.548i$ |
| 0.2 | $0.04 + 4.802 \cdot 10^{-4} i$ | $0.055 - 1.089i$ | $-15.504 - 4.573 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 1.095i$ |
| 0.3 | $0.04 + 7.196 \cdot 10^{-4} i$ | $-0.087 - 1.634i$ | $-15.503 - 6.814 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 1.643i$ |
| 0.4 | $0.04 + 9.591 \cdot 10^{-4} i$ | $-0.287 - 2.179i$ | $-15.501 - 8.996 \cdot 10^{-3} i$ | $-3.511 \cdot 10^3 + 2.191i$ |
| 0.5 | $0.04 + 1.199 \cdot 10^{-3} i$ | $-0.543 - 2.714i$ | $-15.499 - 0.011i$ | $-3.51 \cdot 10^3 + 2.739i$ |
| 0.6 | $0.04 + 1.438 \cdot 10^{-3} i$ | $-0.857 - 3.27i$ | $-15.496 - 0.013i$ | $-3.51 \cdot 10^3 + 3.287i$ |
| 0.7 | $0.04 + 1.678 \cdot 10^{-3} i$ | $-1.228 - 3.816i$ | $-15.493 - 0.015i$ | $-3.51 \cdot 10^3 + 3.836i$ |
| 0.8 | $0.04 + 1.918 \cdot 10^{-3} i$ | $-1.656 - 4.363i$ | $-15.49 - 0.016i$ | $-3.509 \cdot 10^3 + 4.385i$ |
| 0.9 | $0.04 + 2.157 \cdot 10^{-3} i$ | $-2.142 - 4.911i$ | $-15.485 - 0.018i$ | $-3.509 \cdot 10^3 + 4.935i$ |
| 1 | $0.04 + 2.397 \cdot 10^{-3} i$ | $-2.685 - 5.459i$ | $-15.48 - 0.018i$ | $-3.508 \cdot 10^3 + 5.485i$ |

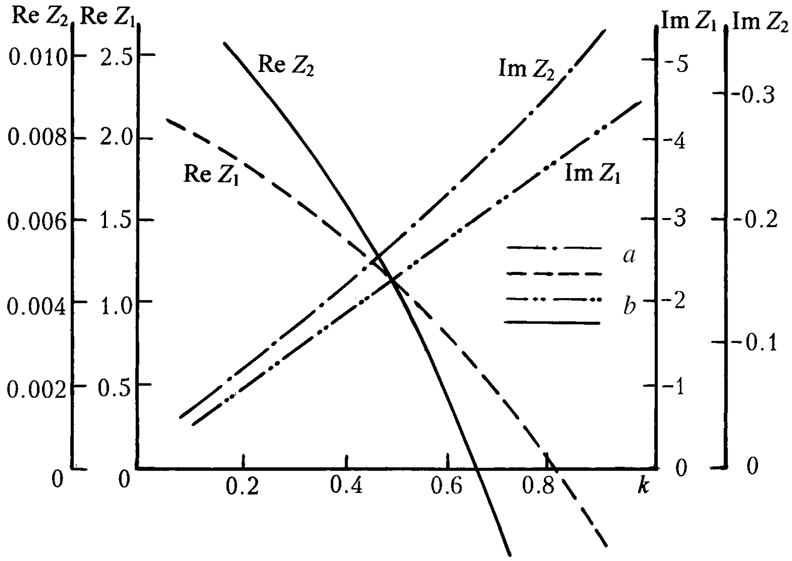


Fig. 1. Dependence of the unstable root on the gradients of disturbances: a) $V = 9$ V, $T_0 = 3500$ K, $P_0 = 3{\cdot}10^7$ Pa; b) $V = 16$ V, $T_0 = 3500$ K, $P_0 = 3{\cdot}10^7$ Pa ($\tilde{x} = 0.0938$ m, $\tilde{t} = 0.844{\cdot}10^{-4}$ sec).

$$\eta_{cr} = \varphi_i \,\text{grad}\, U_s/(\rho\Theta \sqrt{m_a/m_e}) , \qquad (11)$$

below which the instability is not realized.

Using the Saha relation, we can write expression (11) as

$$\left[\text{const}\, \frac{T^{3/2}}{kn_a^{(0)}} \exp\left(-\frac{eV}{kT}\right)\right]^{1/2} \ge \varphi_i \,\text{grad}\, U_s/(\rho\Theta \sqrt{m_a/m_e}) , \qquad (12)$$

wherefrom the stabilizing effect of the ionization potential and the destabilizing effect of the temperature for charge instabilities determined by condition (10) are obvious.

III. For numerical evaluation of conditions (10) we used results of the calculation of flow characteristics from the basic system of equations in the presence of an external sound wave [1].

The estimates were made in the space-time region ($\tilde{x} \in (0.0156–0.0938)$ m and $\tilde{t} \in (0.4–1.2){\cdot}10^{-4}$ sec) for two initial modes:

$$\text{a) } P_0 = 3{\cdot}10^7 \text{ Pa},\ T_0 = 3000 \text{ K},\ V = 9 \text{ V};\ \text{b) } P_0 = 3{\cdot}10^7 \text{ Pa},\ T_0 = 3000 \text{ K},\ V = 16 \text{ V} .$$

The values of the flow characteristics and the spectrum of disturbances at the points $\tilde{x} = 0.0781$ and 0.0938 m at $\tilde{t}_1 = 0.122{\cdot}10^{-3}$ sec (for modes (a) and (b)) and $\tilde{t}_2 = 0.844{\cdot}10^{-4}$ sec (for mode (a)) are given in Table 1.

As the scale parameter $L$ we used the mean free path $\lambda_0 = (\sqrt{2}\, S_{i,e,a} n_a^{(0)})^{-1} = 0.65{\cdot}10^{-8}$ m. The threshold values of the relative content of the electronic component ($\eta = \alpha$ in the case of finite gradients and low velocities $U_i = U_e = U_s$) and the gradient for these modes are respectively equal to (see (12) and Table 1, a and b)

$$\text{a) } \begin{array}{l} \eta = 0.29{\cdot}10^{-7} > \eta_{cr} = 1.91{\cdot}10^{-17} ; \\ k_{cr} = 2.23{\cdot}10^{-1} ,\quad V = 9 \text{ V} , \end{array} \qquad \text{b) } \begin{array}{l} \eta = 0.38{\cdot}10^{-13} > \eta_{cr} = 1.91{\cdot}10^{-17} ; \\ k_{cr} = 2.54{\cdot}10^{-5} ,\quad V = 16 \text{ V} . \end{array} \qquad (13)$$

As is seen from (13), charge instability is possible in both cases, but in case (b) it is possible for lower gradient disturbances. In other words, an increase in the ionization potential considerably decreases the spectrum of disturbances which lead to instability. In case (a), in addition to the flow-charge instability, there also exists the gasdynamic instability which appears locally where the gradient $U_s$ is negative (see Table 1, c). The charge-energy instability (7) is not realized for these modes.

IV. Numerical analysis of the roots of the spectral equation of system (3)

$$\det\,[z\delta_{\alpha\beta} + e_{\alpha\beta}] = Z^4 + A_1 Z^3 + A_2 Z^2 + A_3 Z + A_4 = 0 \tag{14}$$

was made for the points of the above-mentioned space-time region for the two initial modes within the range of variation of $k \in (0–1)$. The results of the calculation are in satisfactory agreement with the dimensionless analysis given above. Depending on the initial conditions, one or two unstable roots corresponding to the gasdynamic (see Table 1, c) and flow instabilities (see Table 1, a–c) exist in the system.

Figure 1 gives the dependences of the unstable root corresponding to the flow-charge instability on the gradients of disturbances.

It follows from Table 1 and Fig. 1 that charge instability develops predominantly on longwave disturbances and decays on shortwave ones. Instabilities in the system of a lower potential of ionization develop much more quickly than in the systems with a high potential (12). It is seen from Fig. 1 that the frequencies of the excited vibrations are proportional to the gradients and correspond to ultrasonic frequencies: $\omega \in (10^{10}–10^{11})$ sec$^{-1}$.

Thus, by analyzing the stability conditions and calculating numerically the characteristics of the spectrum of disturbances we have confirmed that the weakly ionized plasma medium of combustion products in the chambers of thermal energy converters within the assigned range of parameters $V \in (9–16)$ V, $T_0 \sim 10^3$ K, and $P_0 \sim 3{\cdot}10^7$ Pa is unstable at all points of the considered space-time range. This instability manifests itself as the excitation of the fluctuations of pressure, current densities, and charge.

We have obtained explicit conditions of formation of different charge and current instabilities in a flow that have been confirmed by numerical analysis.

Emphasis has been placed upon changes in the direction of the effect of the temperature and the composition of the medium (ionization potential) on stability in transitions from shortwave disturbances to longwave disturbances.

The stabilizing effect of an increase in the ionization potential and the gradients and the destabilizing effect (in terms of an increase in the relative content of the electronic component) of the temperature on longwave disturbances at low velocities of the flow have been noted.

It has been shown that current shortwave instabilities are caused by the competing effects of the temperature (stabilizing) and convection (destabilizing) factors and they can lead to the removal of one charged component with the flow.

## NOTATION

$\tilde{\rho}$, density, kg·m$^{-3}$; $\tilde{p}$, pressure, Pa; $\tilde{T}$, temperature, K; $P_0$ and $T_0$, static components of pressure and temperature of the medium, Pa and K, respectively; $e$, unit electric charge, C; $\tilde{x}$, linear coordinate, m; $\tilde{t}$, time, sec; $S_{(\mathrm{i,e})\mathrm{a}}$, cross section of scattering of carriers on the neutral component, m$^2$; $\tilde{U}^{(0)}_{T_\mathrm{a}}$, initial thermal velocity, m·sec$^{-1}$; $\lambda_{\mathrm{i,e}}$, mean free path of carriers, m; $V$, ionization potential, V; $m$, mass, kg; $L$, spatial scale equal to the mean free path, m; $n^{(0)}_\mathrm{a}$, initial concentration of the neutral component, m$^{-3}$; $\omega$, frequency, sec$^{-1}$; $\alpha$, degree of ionization; $\eta$, relative content of the electronic component; $q$, charge density; $Q$, current density; $E$, strength of the longitudinal electric field; $E_{\mathrm{ind}}$, induced field; $\Theta$, relative energy of the Coulomb interaction; $k$ and $z$, parameters of the Fourier–Laplace transformation. Subscripts: a, e, i, and s, neutral, electron, ion, and medium as a whole; ind, induced; cr, critical. Superscript: ~ , dimensional parameters.

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